研究奨励事業報告書

（理学研究科・研究科長裁量経費）

Daiki Kawabe is interested in the theory of pure motives. The motive of an algebraic variety plays the role of universal Weil cohomology. His results for two themes are as follows.

1. Bloch's conjecture on surfaces(It can be regarded as a special case of conservativity conjecture)

Its motivic statement is:

"Let S be a surface, H(S) its 2^{nd} etale cohomology, and h(S) its Chow motive. If the transcendental part of H(S) vanishes, then the transcendental part of h(S) also vanishes."

To prove the conjecture, he focuses on the motive of a fibered surface, and presents a conjecture:

Conjecture 1

Let $f : X \to \mathbb{P}^1$ be a fibered surface. Assume that $b_2(X) = \rho(X)$ and $b_1(X) = 0$. Then there is a decomposition

$$h(X) \cong h(\mathbb{P}^1) \oplus h(\mathbb{P}^1) \otimes \mathbb{L} \oplus \mathbb{L}^{\oplus r + \sum_{t \in \mathbb{P}^1} (\#\mathrm{Irr}(X_t) - 1)} \quad \text{in} \quad \mathcal{M}(k)_{rat}.$$

Here, $\mathbb{L}$ is the Lefschetz motive, r is the rank of the Jacobian of the generic fiber X_η, and $\mathrm{Irr}(X_t)$ is the set of distinct irreducible components of the closed fiber X_t.

He shows that conjecture 1 is true if and only if the Bloch conjecture is true.

2. Grothendieck's period conjecture (for simplicity, GPC)

Its motivic statement is: "We work over the number field. Let M be a homological motive and c(M) the period isomorphism from the de Rham cohomology of M to the Betti cohomology of M. Let Z(M) be the Zariski closure of c(M) in Isom and Ω(M) the torsor of the motivic period of M. Then the inclusion Z(M) → Ω(M) is equality."

GPC is one of the most difficult problems in the theory of motives. Chudnovsky shows that GPC holds for any CM elliptic curve. This is the only non-trivial example for which GPC is known.

He [1] shows that GPC holds for the Kummer surface Km(A) associated with the abelian surface A isogenous to the self-product E×E of a CM elliptic curve. The point is that its motive h(Km(A)) has a non-trivial transcendental part, but belongs to the Tannakian category <h(E)> generated by the motive of a CM elliptic curve. Kreutz-Shen-Vial [2] show that GPC holds for a K3 surface of co-picard rank 0. This is a generalization of the result [1] (Corollary 9.17 (i) in [2]).

$$\begin{tikzcd} \mathcal{V}(k) \arrow[r, "H^*"] \arrow[d, "h"'] \arrow[dr, phantom, "\circlearrowleft"] & \mathrm{Vect} \\ \mathcal{M}(k) \arrow[ur, dotted, "R_H"'] & \end{tikzcd} \qquad \begin{tikzcd} A \arrow[r, "-id_A"] & A \arrow[d] \\ Km(A) \arrow[r, "birat", "\sim"'] & A/\langle -id_A \rangle \end{tikzcd}$$

[1] D. Kawabe, Grothendieck's period conjecture for Kummer surfaces of self-product CM type, arXiv:2303.05030 [Submitted on 9 Mar 2023].

[2] T. Kreutz, M. Shen, and C. Vial, Around the de Rham-Betti conjecture, arXiv:2206.08618 [Submitted on 17 Jun 2022 (v1), last revised 16 Mar 2023 (v2)].